EQUILIBRIUM STABILITY OF A FLUID LAYER UNDER THE ACTION OF THERMOCAPILLARY FORCES IN THE QUASISTATIONARY APPROXIMATION

L. K. Antanovskii

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The quasistationary approximation is used to describe fluid motion with low accelerations, and consists of discarding inertial forces in the Navier–Stokes equations. This situation is clearly realized in thin fluid layers, capillaries, and small-size drops, when surface tension forces dominate over mass-weight forces included in the inertial terms.

The problem considered here is that of equilibrium stability of a fluid layer at the surface of a circular cylinder with an isothermal free boundary. Based on representing solutions of the Stokes system in terms of the bianalytic stress–current function, the elliptic part of the problem is reduced to a system of one-dimensional Fredholm integral equations of the second kind. Following its solution, the nonstationary kinematic condition leads to a parabolic pseudo-differential equation in the perturbation parametrization of the free boundary. The spectrum of the linearized "normal velocity" is calculated, and a stability condition is obtained for the cylindrical shape of the free boundary with respect to small planar initial perturbations.

1. Statement of the Problem. Let a viscous, incompressible fluid undergo quasistationary plane-parallel motion under the action of thermocapillary forces. We denote by $\Omega = \Omega(t)$ a doubly-connected region, filled by a fluid at the moment of time t, with a fixed cylindrical wall Σ and a free boundary $\Gamma = \Gamma(t)$. The mathematical formulation of the problem consists of finding the flow region Ω, the velocity $v = v_x + iv_y$, and the pressure p as functions of the point $z = x + iy \in \Omega$ and of time t, satisfying the equations

$$\nabla p = \mu\Delta v,\ \operatorname{div} v = 0 \text{ in } \Omega; \tag{1.1}$$

$$v = 0 \text{ on } \Sigma; \tag{1.2}$$

$$P(n) = (d/ds)(\sigma dz/ds) \text{ on } \Gamma; \tag{1.3}$$

$$V_n = v\cdot n \text{ on } \Gamma; \tag{1.4}$$

$$\Gamma = \Gamma_0 \text{ for } t = 0. \tag{1.5}$$

Here $P(n) = pn - \mu(n\cdot\nabla v + \nabla v\cdot n)$ is the pressure vector (impulse flow) along the internal normal $n = idz/ds$; s is the arc length of the boundary Ω, μ is the dynamic viscosity coefficient, σ is the varying surface tension coefficient, and V_n is the displacement velocity of Γ along the normal n.

By the well-known Frenet equation $d^2z/ds^2 = kn$ (k is the curvature of Γ) the dynamic condition (1.3) acquires the standard form (in equilibrium we have the Laplace equation $p = \sigma k$). In the general case a tangential component of the pressure vector is generated on Γ, equal to $d\sigma/ds$, and leading to fluid convection. In the problem of thermocapillary convection the dependence of σ on temperature θ is assumed known: $\sigma = \sigma(\theta)$. For simplicity it is assumed that θ satisfies the quasistationary problem

$$\Delta\theta = 0 \text{ in } \Omega,\ \theta = \theta_0 \text{ on } \Sigma,\ d\theta/dn = \beta(\theta - \theta_\infty) \text{ on } \Gamma \tag{1.6}$$

(θ_0 is the temperature of the wall Σ, θ_∞ is the temperature of the adjacent gas, and β is the interphase heat exchange coefficient).

Below we consider the specific problem in which the wall $\Sigma = \{|z| = R_0\}$ has a constant temperature θ_0, not equal to θ_∞. There exists then an equilibrium state of the fluid layer with an isothermal free boundary $\Gamma = \{|z| = R\}$ (for definiteness $R > R_0$). In this case the solution of (1.6) is

$$\theta(z) = \theta_0 + (\theta_\infty - \theta_0)B\ln(|z|/R_0)/(1 + aB),$$

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where $a = \ln(R/R_0)$ and $B = \beta R$ is the Biot number. From the free boundary temperature $\theta_* = (\theta_0 + aB\theta_\infty)/(1 + aB)$ one determines the thermocapillary number

$$C = \frac{B(\theta_0 - \theta_\infty)\sigma'(\theta_*)}{(1 + aB)\sigma(\theta_*)} \equiv R\frac{d\theta}{dn}\frac{\sigma'(\theta)}{\sigma(\theta)} \text{ on } \Gamma,$$

whose sign also determines the stability. More precisely, it will be shown that the equilibrium state described is stable with respect to a small initial perturbation Γ_0 if $C > 0$, and unstable when the opposite inequality $C < 0$ is satisfied. For purely free boundaries $\sigma'(\theta) < 0$, therefore the instability starts at $\theta_0 > \theta_\infty$.

The following scheme is suggested for solving the problem (1.1)-(1.6). We fix the curve Γ at moment of time t, and solve the auxiliary problem (1.1)-(1.3), (1.6) in the known region Ω. Since this problem is uniquely solvable, one determines the "normal velocity" operator $N(\Gamma)$, comparing the curve Γ with the normal velocity component $v \cdot n$ on Γ [1]. Equations (1.4)-(1.5) are now written down formally in the form of the Cauchy problem for the curve Γ ($\partial\Gamma/\partial t = V_n$):

$$\partial\Gamma/\partial t = N(\Gamma),\ \Gamma(0) = \Gamma_0. \tag{1.7}$$

Obviously, all stability information within the linear approximation of fluid equilibrium is included in the structure of the spectrum of the operator $N(\Gamma)$ linearized at rest.

2. The Complex Stress–Current Function. To solve the auxiliary problem it is convenient to transform to complex variables. We introduce the stream function ψ by the equalities $v_x = -\partial\psi/\partial y$, $v_y = \partial\psi/\partial x$ or $v = i\nabla\psi$. From (1.1) follows the equation $\Delta^2\psi = 0$, therefore, $\psi = \operatorname{Im} w$, where $w = \varphi + i\psi$ is a bianalytic function: $\partial^2 w/\partial \bar{z}^2 = 0$ ($2\partial/\partial z = \partial/\partial x - i\partial/\partial y$, $2\partial/\partial\bar{z} = \partial/\partial x + i\partial/\partial y$ are the Cauchy–Riemann operators). In analogy with planar elasticity theory, the function $\varphi = \operatorname{Re} w$ is called the stress function, while w is the stress–current function. For nonvortical motion φ transforms into the velocity potential: $v = -\nabla\varphi$. It is easily established that the stress function is determined from the stream function accurately within an arbitrary term

$$c_0(x^2 + y^2) + c_1 x + c_2 y + c_3. \tag{2.1}$$

Since the function Δw is analytic, by the Cauchy–Riemann conditions we have from the Stokes system (1.1)

$$\frac{dp}{ds} = -\mu\frac{d}{dn}(\Delta\psi) = -\mu\frac{d}{ds}(\Delta\varphi).$$

Due to the arbitrariness of the number c_0, it can be assumed in expression (2.1) that $p = -\mu\Delta\varphi$; therefore,

$$P(n) = -\mu(\Delta\varphi)n - 2\mu(\partial v/\partial\bar{z})\bar{n} = -4\mu i\left\{\frac{\partial^2\varphi}{\partial z\partial\bar{z}}\frac{dz}{ds} - i\frac{\partial^2\psi}{\partial\bar{z}^2}\frac{d\bar{z}}{ds}\right\} = -4\mu i\left\{\frac{\partial^2\varphi}{\partial z\partial\bar{z}}\frac{dz}{ds} + \frac{\partial^2\varphi}{\partial\bar{z}^2}\frac{d\bar{z}}{ds}\right\} = -4\mu i\frac{d}{ds}\left(\frac{\partial\varphi}{\partial\bar{z}}\right).$$

Consequently, the Kolesov–Muskhelshvili representation is valid

$$P\left(i\frac{dz}{ds}\right) = -2\mu i\frac{d}{ds}(\nabla\varphi),\quad v = i\nabla\psi,\quad p = -\mu\Delta\varphi. \tag{2.2}$$

In terms of $w = \varphi + i\psi$, and with the use of (2.2), the boundary conditions (1.2)-(1.3) are written in symmetric form

$$\begin{aligned} &\varphi = 0,\ 2\mu d\varphi/dn = \sigma(\theta) && \text{at } \Gamma,\\ &\psi = 0,\ d\psi/dn = 0 && \text{at } \Sigma. \end{aligned} \tag{2.3}$$

In integrating the dynamic condition (1.3) we used the arbitrariness of the numbers c_1, c_2, c_3 in (2.1). Following the solution of problem (1.6), (2.3) with a fixed curve Γ, the "normal velocity" operator is calculated as $N(\Gamma) = d\psi/ds$.

3. Mixed Problem for the Bianalytic Function. For the bianalytic function $w = \varphi + i\psi$ in a doubly-connected region Ω, restricted by the closed curves Γ and Σ, we consider the

following mixed problem of general form

$$\varphi = f_1, \ d\varphi/dn = dg_1/ds \quad \text{on } \Gamma, \qquad \psi = f_2, \ d\psi/dn = dg_2/ds \quad \text{on } \Sigma \tag{3.1}$$

(f_1, f_2, dg_1/ds, dg_2/ds are given functions).

The Hoersch representation $w(z) = w_0(z) + \bar{z}w_1(z)$ is valid, where $w_0(z)$, $w_1(z)$ are analytic functions, in terms of which problem (3.1) is written as follows:

$$\begin{aligned} &\operatorname{Re}(w_0 + \bar{z}w_1) = f_1, \quad 2\operatorname{Im}\left(w_1 \frac{d\bar{z}}{ds}\right) - \frac{d\Psi}{ds} = \frac{dg_1}{ds} \quad \text{on } \Gamma, \\ &\operatorname{Im}(w_0 + \bar{z}w_1) = f_2, \quad 2\operatorname{Re}\left(w_1 \frac{d\bar{z}}{ds}\right) - \frac{d\varphi}{ds} = -\frac{dg_2}{ds} \quad \text{on } \Sigma. \end{aligned} \tag{3.2}$$

In fact, due to the equality $dz/dn = n = idz/ds$, we obtain $dw_0/dn = idw_0/ds$, $dw_1/dn = idw_1/ds$; therefore, $dw/dn = idw/ds + 2\bar{n}\partial w/\partial \bar{z}$.

We write down initially a solution of the mixed problem for the analytic function $\Phi(\tau)$ in the canonical region

$$G_\alpha = \{0 < \operatorname{Im}\tau < \alpha\}/2\pi Z$$

(by definition, the functions analytic in the "ring" G_α are 2π-periodic and analytic functions in the strip $\{0 < \operatorname{Im}\tau < \alpha\}$). More precisely, the problem $\operatorname{Re}\Phi(\lambda) = F_1(\lambda)$, $\operatorname{Im}\Phi(\lambda + i\alpha) = F_2(\lambda)$ has a unique solution ($\tau_0 \in G_\alpha$)

$$\Phi(\tau_0) = \frac{1}{2\pi i}\int_{\partial G_\alpha} M(\tau - \tau_0) F(\tau)\, d\tau \equiv \frac{1}{2\pi i}\int_0^{2\pi} M(\lambda - \tau_0) F_1(\lambda)\, d\lambda - \frac{1}{2\pi}\int_0^{2\pi} M(\lambda + i\alpha - \tau_0) F_2(\lambda)\, d\lambda,$$

where $F(\lambda) = F_1(\lambda)$, $F(\lambda + i\alpha) = iF_2(\lambda)$, and $M(\tau) = \cot\frac{\tau}{2} - 2\sum_{k=1}^{\infty} e^{-k\alpha}\frac{\sin k\tau}{\operatorname{ch} k\alpha}$. We similarly define the operator

$$S\colon \{\operatorname{Re}\Phi(\lambda), \ \operatorname{Im}\Phi(\lambda + i\alpha)\} \mapsto \{\operatorname{Im}\Phi(\lambda), \ -\operatorname{Re}\Phi(\lambda + i\alpha)\}$$

with the singular integral

$$\begin{aligned} &\mathbf{S}(\mathbf{f} \mid \lambda_0) = \frac{1}{2\pi}\int_0^{2\pi} S(\lambda_0 - \lambda)\cdot \mathbf{f}(\lambda)\, d\lambda, \quad \mathbf{f} = \{f_1, f_2\}, \\ &S(\lambda) = \begin{pmatrix} M(\lambda) & M_*(\lambda) \\ -M_*(\lambda) & -M(\lambda) \end{pmatrix}, \quad M_*(\lambda) = iM(\lambda + i\alpha) = \sum_{k=-\infty}^{\infty} \frac{\cos k\lambda}{\operatorname{ch} k\alpha}. \end{aligned} \tag{3.3}$$

Let the analytic 2π-periodic function $z = z(\tau)$ satisfy the conformal mapping G_α on Ω with norm $z(i\alpha) = R_0 e^{i\beta}$. It is well known that the conformal invariant α is uniquely determined in the region Ω [2]. Following the variable replacement $z = z(\tau)$, problem (3.2) acquires the form

$$\varphi(\lambda) \equiv \operatorname{Re}\{w_0(\lambda) + \overline{z(\lambda)}w_1(\lambda)\} = f_1(\lambda), \tag{3.4}$$

$$\Psi(\lambda + i\alpha) \equiv \operatorname{Im}\{w_0(\lambda + i\alpha) + \overline{z(\lambda + i\alpha)}w_1(\lambda + i\alpha)\} = f_2(\lambda);$$

$$\begin{aligned} &2\operatorname{Re}\left\{\frac{w_1(\lambda)}{iz'(\lambda)}\right\} = \frac{g_1'(\lambda) + \Psi'(\lambda)}{|z'(\lambda)|^2}, \\ &2\operatorname{Im}\left\{\frac{w_1(\lambda + i\alpha)}{iz'(\lambda + i\alpha)}\right\} = \frac{g_2'(\lambda) - \varphi'(\lambda + i\alpha)}{|z'(\lambda + i\alpha)|^2} \end{aligned} \tag{3.5}$$

(the functional notational are the same as earlier, and the prime corresponds to differentiation with respect to λ).

We introduce the vector $\gamma(\lambda) = \{\Psi(\lambda), -\varphi(\lambda + i\alpha)\}$ and obtain for it an integral equation. For this purpose, from (3.5) and for given right hand sides we represent $w_1(\tau)$ in terms of $\gamma(\lambda)$ and substitute it into Eq. (3.4). From (3.4) we find $w_0(\tau)$, following which we calculate the vector γ in terms of w_0, w_1, obtaining

$$\boldsymbol{\gamma} = \mathbf{S}(\mathbf{f}) + \mathbf{T}(\mathbf{g}) + \mathbf{T}(\boldsymbol{\gamma}). \tag{3.6}$$

Here $\mathbf{T}(\boldsymbol{\gamma}\,|\,\lambda_0) = -\frac{1}{2\pi}\int\limits_0^{2\pi} T(\lambda_0, \lambda)\cdot d\boldsymbol{\gamma}(\lambda)$;

$$T(\lambda_0, \lambda) = \begin{pmatrix} \operatorname{Re} K(\lambda_0, \lambda) & \operatorname{Im} K(\lambda_0, \lambda + i\alpha) \\ \operatorname{Im} K(\lambda_0 + i\alpha, \lambda) & -\operatorname{Re} K(\lambda_0 + i\alpha, \lambda + i\alpha) \end{pmatrix};$$

$$K(\tau_0, \tau) = \frac{1}{4\pi\,|z'(\tau)|^2} \int\limits_{\partial G_\alpha} \{[\overline{z(\tau_0)} - \overline{z(\xi)}]\, M(\tau_0 - \xi) -$$

$$[\overline{z(\tau_0)} - \overline{z(\tau)}]\, M(\tau_0 - \tau)\}\, M(\xi - \tau)\, dz(\xi).$$

In calculating the operator $\mathbf{T}$ it is necessary to use the identity

$$w_1(\tau_0) = \frac{1}{2\pi i} \int\limits_{\partial G_\alpha} M(\tau - \tau_0)\, w_1(\tau)\, d\tau, \quad \tau_0 \in \partial G_\alpha.$$

It can be shown that the kernel $dT(\lambda_0, \lambda)/d\lambda_0$ has a weak singularity if Γ and Σ are Lyapunov curves, i.e., $z'(\tau)$ belongs to the Hölder class. Equation (3.6) is a Fredholm integral equation of the second kind, therefore it is always solvable due to the uniqueness of the vanishing solution of the homogeneous auxiliary problem (1.1)-(1.3). On the function $z = e^{i\tau}$, satisfying the mapping G_α in the annulus $\{e^{-\alpha} < |z| < 1\}$, the kernel $T(\lambda_0, \lambda)$ is calculated explicitly

$$T(\lambda_0, \lambda) = \begin{pmatrix} 0 & e^{\alpha} \\ e^{-\alpha} & 0 \end{pmatrix} \operatorname{sh} \alpha M_*(\lambda_0 - \lambda). \tag{3.7}$$

Obviously, the kernels K and T do not change when $z(\tau)$ is replaced by $az(\tau) + b$.

4. Parametrization of the Free Boundary. We transform problem (1.7) to an integrodifferential equation in some parametrization of the curve Γ. In our specific situation it is convenient to represent the mapping $z(\tau)$ in terms of the real 2π-periodic function $\eta(\lambda) = \ln(|z(\lambda)|/R)$. For this it is necessary to solve the Schwartz problem

$$\operatorname{Re}\ln\left\{\frac{z(\lambda)}{Re^{i\lambda}}\right\} = \eta(\lambda), \quad \operatorname{Re}\ln\left\{\frac{z(\lambda + i\alpha)}{Re^{\,i(\lambda + i\alpha)}}\right\} = \alpha - a, \tag{4.1}$$

whose solvability condition leads to an expression for the conformal invariant

$$\alpha = a + \frac{1}{2\pi}\int\limits_0^{2\pi} \eta(\lambda)\, d\lambda. \tag{4.2}$$

The function $z(\lambda)$ in $\eta(\lambda)$ is reconstructed from (4.1) by means of the Hilbert transformation $\mathbf{H}$ in the form

$$z(\lambda) = R\exp\{i\lambda + \eta(\lambda) + i\mathbf{H}(\eta|\lambda)\},$$
$$\mathbf{H}(\eta\,|\,\lambda_0) = \frac{1}{2\pi}\oint\limits_0^{2\pi} H(\lambda_0 - \lambda)\,\eta(\lambda)\, d\lambda, \quad H(\lambda) = \operatorname{ctg}\frac{\lambda}{2} + 2\sum_{k=1}^{\infty} e^{-k\alpha}\frac{\sin k\lambda}{\operatorname{sh} k\alpha}. \tag{4.3}$$

The mapping constant is pure imaginary due to the normalization $z(i\alpha) = R_0 e^{i\beta}$ (the angle β is unimportant).

Following simple calculations with the use of (4.3), Eq. (1.7) or

$$\operatorname{Im}\{\bar{z}'\partial z/\partial t\} = \Psi'$$

in terms of η acquires the form

$$\{1 + H(\eta')\}\frac{\partial\eta}{\partial t} - \eta'\frac{\partial H(\eta)}{\partial t} = -\frac{\Psi'}{R^2 e^{2\eta}}. \tag{4.4}$$

We note that the Hilbert kernel $H(\lambda)$ depends on time t in terms of the number α, related to η by Eq. (4.2) (obviously, the kernel $\partial H/\partial\alpha$ is nonsingular). The function $\Psi(\lambda)$ is determined at each moment of time by the conformal mapping $z(\tau)$ as the first component of the solution $\Upsilon(\lambda)$ of Eq. (3.6).

5. Linearization of the Problem. Using the conformal mapping $z(\tau)$ and the kernel $M(\tau)$, the mixed problem of Eq. (1.6) for the temperature can be reduced to a one-dimensional integral equation. We restrict ourselves here to writing the expression for the linear part of the solution in the perturbation $\eta(\lambda)$

$$\sigma(\theta) = \sigma(\theta_*)\{1 - C\mathbf{A}(\eta\,|\,\lambda)\} \quad \text{for} \quad z = z(\lambda). \tag{5.1}$$

Here the operator $\mathbf{A}$ is

$$\widehat{A}_k = \frac{1 + \operatorname{th} ka/k}{1 + B \operatorname{th} ka/k}, \quad \widehat{A}_0 = \frac{1+a}{1+Ba},$$

i.e., it acts on the function $\eta(\lambda)$ according to the following equation:

$$\mathbf{A}(\eta\,|\,\lambda) = \sum_k \widehat{A}_k \widehat{\eta}_k e^{ik\lambda}, \quad \eta(\lambda) = \sum_k \widehat{\eta}_k e^{ik\lambda}.$$

To linearize problem (3.6) it is convenient to supplement the stress function $\Phi(z)$ by the function $\sigma(\theta_*)(|z|^2 - R^2)/(4\mu R)$ of shape (2.1), with $\Phi = \Psi = 0$ on the basis of the solution $\eta = 0$, and accurately within quadratic terms we obtain the expressions

$$2\mu f(\lambda) = \sigma(\theta_*) R\{\eta(\lambda), 0\},$$
$$2\mu g'(\lambda) = -\sigma(\theta_*) R\{\eta(\lambda) + C\mathbf{A}(\eta\,|\,\lambda), 0\}.$$

Using now (3.3), (3.7), (4.2), and (5.1), the linear problem in $\eta(\lambda)$ (3.6) is solved, and following substitution of the first component of $\Upsilon(\lambda)$ into the linearized equation (4.4), the following problem is generated

$$\partial\eta/\partial t = -[\sigma(\theta_*)/(2\mu R)]\,\mathbf{L}(\eta), \tag{5.2}$$

where $\mathbf{L}(\eta)$ is a pseudodifferential operator with real argument

$$\widehat{L}_k = k \operatorname{th} ka \frac{1 + (C\widehat{A}_k \operatorname{th} a - 1)\, k \operatorname{sh} 2a/\operatorname{sh} 2ka}{1 + (k \operatorname{sh} a/\operatorname{ch} ka)^2}. \tag{5.3}$$

For asymptotic stability of the vanishing solution in the perturbation η at t = 0 it is necessary to satisfy the inequalities $\hat{L}_k > 0$ for $k \geq 1$, which is equivalent to the condition $C > 0$. On the other hand, for $C < 0$ we clearly have $\hat{L}_1 < 0$. Thus, the sign of the thermocapillary number completely determines the state stability.

It is seen from (5.3) that $\hat{L}_k \sim |k|$ for $k \to \infty$, i.e., the pseudodifferential operator $\mathbf{L}$ is of first order. Consider the asymptotic symbol (5.3) for $a \to 0$, restricting k to $C = C_0 a$ (C_0 = const), corresponding to the long-wave approximation of a thin layer. In the given case the expansion

$$\widehat{L}_k = \{2(k^2 - 1)/3 + C_0\}k^2 a^3 + O(a^4)$$

can be associated with the contracted fourth-order differential equation (5.2)

$$\frac{\partial\eta}{\partial t} + \frac{\sigma(\theta_*)\,a^3}{\mu R}\left\{\frac{1}{3}(\eta'' + \eta) - \frac{1}{2}C_0\eta\right\}'' = 0.$$

It finally coincides with the Reynolds equation, linearized in dimensionless variables, of lubrication theory applied to thermocapillary motion of a fluid layer of thickness h at the wall $\Sigma = \{|z| = R_0\}$ [3]

$$\frac{\partial h}{\partial t} + \operatorname{div}_\Sigma \left\{ \frac{\sigma_*(h)\, h^3}{3\mu} \nabla_\Sigma \left(\Delta_\Sigma h + \frac{h}{R_0^2} \right) + \frac{h^2}{2\mu} \nabla_\Sigma \sigma_*(h) \right\} = 0.$$

The dependence $\sigma_*(h) = \sigma((1 - \beta h)\theta_0 + \beta h\theta_\infty)$ is obtained here as a result of asymptotic integration of the heat conduction equation for $h/R_0 \to 0$. The linearization is carried out in the constant layer thickness $h = R - R_0$.

In conclusion we note that the critical thermocapillary numbers, making the operator $\hat{L}_k$ vanish, were obtained in [4]. The branching of stationary solutions of the complete equations of thermocapillary convection was established in [5] near the critical Marangoni numbers. These numbers were calculated in [6] for a nondeformed free boundary.

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THEORETICAL AND EXPERIMENTAL STUDY OF CONVECTION IN A LIQUID LAYER WITH LOCAL HEATING

V. A. Al'vares-Suares, Yu. S. Ryazantsev, and V. M. Shevtsova

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The existence of shearing forces associated with surface tension at phase boundaries (liquid–liquid, liquid–gas) may have a significant effect on heat and mass transfer in a liquid. In the case where a temperature gradient is created in the volume of liquid being studied, surface thermocapillary forces – due to their low inertia – may lead to the development of fast-moving hydrodynamic flows [1, 2]. These effects become particularly important in space technology in connection with the study of the behavior of materials (melts) under low-gravity conditions, when the role of thermogravitational convection becomes negligibly small [3]. Possible applications here include crystal growth, welding, and the production of foamed materials in space.

The phenomenon of thermocapillary convection (TCC) (Marangoni effect) makes some contribution to mass transfer in normal production processes as well. In the laser treatment of the surface of metals, TCC may play an important role in the alloying and nitriding of different grades of steel [4]. With allowance for the change in the form of the surface under the influence of TCC, possible uses of TCC include the production of diffraction gratings [5] and a new type of photographic process called thermoextensography [6]. This

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